All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

Graded

Exercise 1. Let P, Q be logical statements. Prove that the statement $\neg(P \wedge Q) \iff (\neg P \vee \neg Q)$ is true. Relate this to a set theoretic law.

Exercise 2. Let P, Q be logical statements. Prove the statement $(P \Rightarrow Q) \iff (\neg Q \Rightarrow \neg P)$.

Exercise 3. Prove the distributive laws and DeMorgan's laws for sets A, B, C:

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
3. $A - (B \cap C) = (A - B) \cup (A - C)$
4. $A - (B \cup C) = (A - B) \cap (A - C)$

N.B. After having proven the distributive laws for sets, we get as a consequence of the way we defined sets a similar law for predicates: $P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$, etc. The proof of each follows immediately from Exercise 3. It is worth understanding these relationships between sets and logic.

Non-graded

Exercise 4. Write the contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$," and determine which (if any) of these statements are true.

Exercise 5. Given sets A, B, C, express each of the following sets in terms of A, B, C and the symbols $\cup, \cap, -$

1. $D = \{x : x \in A \wedge (x \in B \vee x \in C)\}$
2. $E = \{x : (x \in A \wedge x \in B) \vee x \in C\}$

3. $F = \{x : x \in A \wedge (x \in B \Rightarrow x \in C)\}$.

Exercise 6. Let A be a set with n elements. How many elements does the power set $\mathscr{P}(A)$ have. Recall the elements of $\mathscr{P}(A)$ are subsets of A.

Exercise 7. Formulate DeMorgan's laws for arbitrary unions and arbitrary intersections.